Development of Energy Finite Element Analysis in Vibration Analysis of Composite Laminate Plate Structures

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Keywords: Energy Finite Element Analysis, High Frequency Vibration, Energy Transmission, Composite Laminate Plate, Energy Density.

Abstract. The high frequency vibration analysis of a composite laminate plate structure subjected to impact loads was investigated by using method of energy finite element analysis (EFEA). The time and space averaged energy density was used as the primary variable to form the governing differential equations. The multilayer laminate plate is simplified to be equivalent isotropic plate using the average concept, such as the average damping loss factor and the average group speed. The global system of EFEA equations can be solved numerically and the energy density distribution within the whole system can then be obtained. The EFEA numerical results for composite laminate plate structure are validated through comparison with those of very dense conventional finite element analysis (FEA).

Introduction

Fiber-reinforced composite materials have been widely used in high-performance structures that need to be lightweight, yet strong enough to take harsh loading conditions, such as car, ship, and spacecraft [1]. The innovative designs and the increasingly demanding performance requirements make it very importance to address the vibration issues in the design phase.

To improve the performance of composite materials and utilize their advantages significantly, it is crucial to have an effective prediction of their response subjected to impact loads in practical engineering problems. The conventional FEA requires a large number of elements to capture the high frequency characteristics of the structures, which results in very long computational time and high costs. The EFEA formulation has been developed for making an accurate prediction of vibration response at frequencies where conventional FEA are no longer efficient [2].

EFEA method is a new method for simulating vibration response at high frequencies. A power flow finite element analysis is developed by Nefske and Sung [3]. Bouthier and Bernhard[4] derived the equations of time and space averaged energy density and intensity in the far-field. In other research, the EFEA formulation was applied to beams, membranes, plates and acoustic cavities[5]. While most of the research works of EFEA are related to isotropic materials, this paper aims to develop the EFEA formulation for computing the vibration response of the structures of composite laminate plates [6]. The global system of EFEA equations can be solved numerically and the energy density distribution within the entire system can be obtained with relatively small amount of mesh. Therefore, the vibration response of large scale composite laminate plate structures can be predicted in the design process efficiently.

Energy Governing Differential Equations for Infinite Plates

The EFEA formulation uses the time and space averaged energy density as the primary variable to form the governing differential equations. The expressions which govern the flow of energy in an infinite transversely vibrating plate were derived. The flux of energy across given closed surface,

using the control volume approach, is equivalent to the rate of change of the total energy inside the surface which encloses the volume.

$$\partial e/ \partial t = P_{in} - P_{diss} - \nabla \cdot \vec{I}. \tag{1}$$

where e is the energy density, P_{in} is the input power density, P_{diss} is the power density dissipated, ∇ is the del operator, $\vec{I}$ is energy intensity. Eq. 1 is an energy balance relationship for all elastic media and is valid for transient or steady analysis.

On a time averaged basis, the potential energy and the kinetic energy density are approximately equal in the far-field. The energy governing equation in the infinite plate is

$$- (c_g^2 / \eta\omega) (\partial^2\bar{e}/ \partial x^2 + \partial^2\bar{e}/ \partial y^2) + \eta\omega\bar{e} = P_{in}. \tag{2}$$

where c_g is the group speed, η is the damping loss factor, ω is the circular frequency. The key assumptions used in the derivation including harmonic excitation, steady state analysis, far-field approximation and symmetry about the point of excitation. A finite element formulation is developed to solve the Eq. 2 numerically.

$$[\boldsymbol{K_e}]\ \{e\} = [\boldsymbol{Q_e}] + [\boldsymbol{F_e}]. \tag{3}$$

where $[\boldsymbol{K_e}]$ is the system matrix for each element, $\{e\}$ is the vector of nodal value for the time and space averaged energy density, $[\boldsymbol{Q}_e]$ is the power flow across the element boundary, $[\boldsymbol{F}_e]$ is the excitation vector, which represents the input energy at each node.

Equivalent Isotropic Material for Composite Laminate Plates

Averaged Group Speed. Group speed of transverse wave propagation for isotropic materials can be expressed as

$$c_g = 2(D\omega^2/\rho h)^{1/4}. \tag{4}$$

where $D = Eh^3/[12(1-\nu^2)]$, E is the Young's modulus, h is the thickness of the plate, ρ is the density of material, ν is the Poisson's ratio. For the composite laminate plate, the group speed is a function of wave propagation angle α. Removing the dependency on angle α by taking the average c_g from 0 to 2π, the averaged group speed $\bar{c}_g$ can be expressed as

$$\bar{c}_g = \int_0^{2\pi} c_g(\alpha_i)d\alpha_i \,/ \int_0^{2\pi} d\alpha_i. \tag{5}$$

Averaged Damping Loss Factor. The total time-averaged energy loss associated with an arbitrary wave type can be derived using hysteretic damping model applied to each layer

$$\langle \bar{P}_{diss} \rangle = \sum_{l=1}^{n} \eta_l \omega \langle e \rangle_l\ . \tag{6}$$

where η_l *and* $\langle e \rangle_l$ are the damping loss factor and energy density of the l-th layer of the multi-layered composite plate respectively. Since $\langle e \rangle = \langle T \rangle + \langle V \rangle = 2\langle V \rangle$, where $\langle T \rangle$ and $\langle V \rangle$ are the time-averaged total kinetic and potential energies, $\langle T \rangle$ and $\langle V \rangle$ are given by

$$\langle T \rangle = (\omega^2/4)\int_V \rho\ \mathbf{u}^{\mathrm{T}}\boldsymbol{u}d\mathrm{V} = \omega^2\mathbf{u}^{\mathrm{T}}\mathbf{M}\boldsymbol{u},\quad \langle V \rangle = (1/4)\int_V \boldsymbol{\sigma}^{\mathrm{T}}\varepsilon d\mathrm{V} = \boldsymbol{u}^{\mathrm{T}}\mathbf{K}\boldsymbol{u}. \tag{7}$$

where $\boldsymbol{u}$ is the vector of the nodal degrees of freedom, $^{\mathbf{T}}$ stands for Hermitian transpose. **M** and **K** are, respectively, the mass matrices and stiffness. The circular frequency for wave number k can be expressed in terms of the Rayleigh quotient, $\omega^2 = \boldsymbol{u}^{\mathbf{T}}\mathbf{K}(k)\boldsymbol{u} \,/\, \boldsymbol{u}^{\mathbf{T}}\mathbf{M}\boldsymbol{u}$. The damping loss factor associated with each type propagating wave can be expressed as

$$\eta = \sum_{l=1}^{n} \eta_l \, \boldsymbol{u}^{\mathbf{T}}\mathbf{K}^{(l)}\boldsymbol{u} \,/\, (\boldsymbol{u}^{\mathbf{T}}\mathbf{K}\boldsymbol{u}). \tag{8}$$

where $\mathbf{K}^{(l)}$ is the stiffness matrix of the l-th layer. Eq. 8 is derived for each wave heading angle α_i, and the angle averaged damping loss factor must be considered to account for the full range of wave propagation directions from 0 to 2π.

$$\bar{\eta} = \int_0^{2\pi} \eta(\alpha_i) d\alpha_i \,/\, \int_0^{2\pi} d\alpha_i. \tag{9}$$

Simulation and Verification

Using the averaged group speed and averaged damping loss factor, the EFEA differential equation for composite laminate plate can be expressed as

$$-(\bar{c}_g^{\,2} / \bar{\eta}\omega)\,(\partial^2\bar{e}/\partial x^2 + \partial^2\bar{e}/\partial y^2) + \bar{\eta}\omega\bar{e} = \bar{P}_{in}. \tag{10}$$

From the averaged group speed $\bar{c}_g$, the equivalent isotropic material property of composite laminate plate can be defined using equation $\bar{D} = \bar{c}_g^{\,4}\rho h/16\omega^2$, where ρ is density and h is the thickness of the plate. The EFEA capability for modeling and calculating the response of composite laminate plates in high frequencies is validated by comparing simulation results computed by EFEA with those of the very dense FEA.

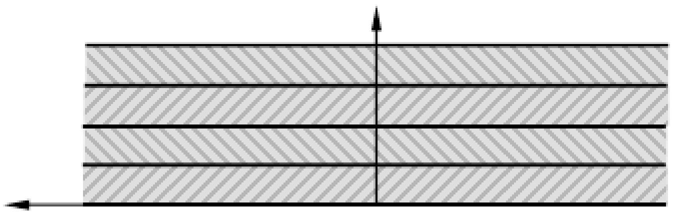


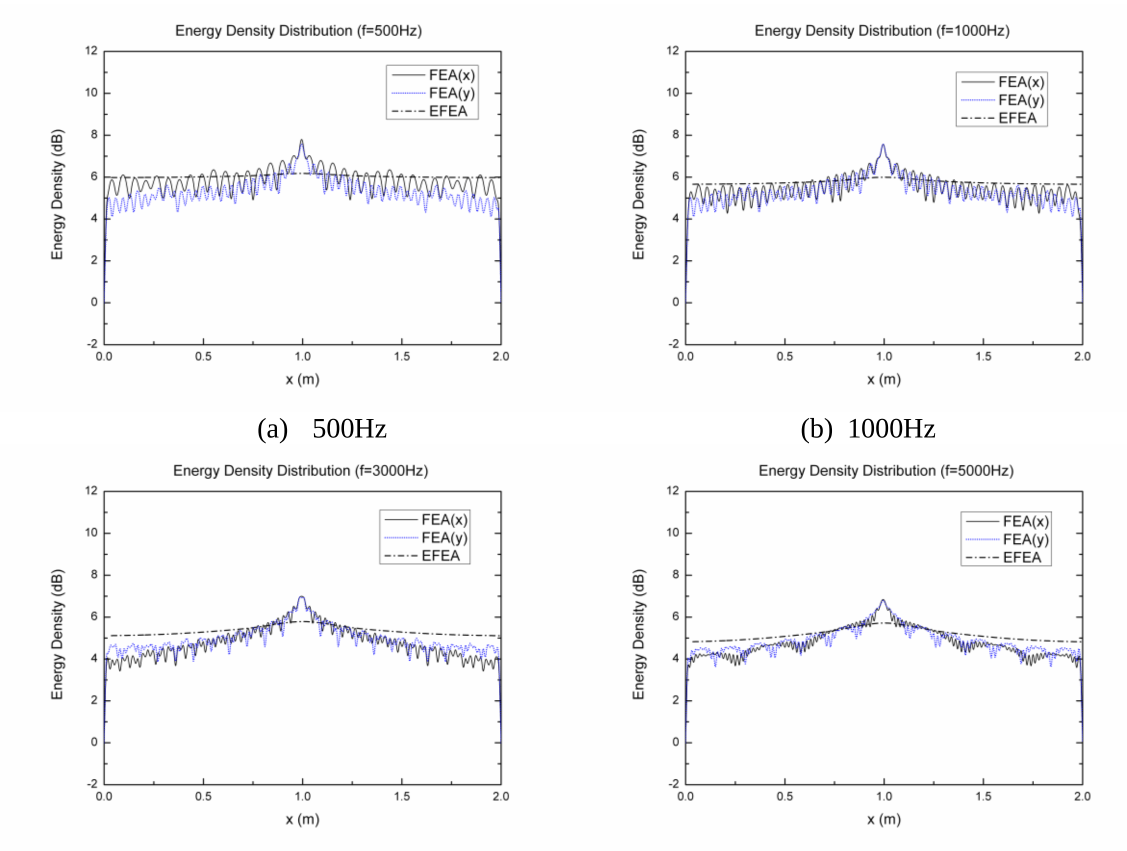
Fig. 1. Schematic diagram of a 4-layer composite laminate plate (30°/45°/-45°/-30°).

The plate is a 2 m × 2 m square laminated plate with four layers of equal thickness 0.25e-3 m. The four layers are at 30°/45°/-45°/-30° orientations. The engineering constants of layers are

E_{11}=37.78e9[Pa], E_{22}=10.0e9[Pa], E_{33}=10.0e9[Pa].
G_{12}=G_{23}=G_{31}=4.91e9 [Pa], ν_{12}=0.25, ρ=1777.6 [kg/m^3].

A harmonic force was applied at the center of the plate, and the input power was 1e-4 *Watt*. The element numbers of EFEA model are 100, while the element numbers of the very dense FEA model are 10,000. The computations were, respectively, carried out using the self-developed program and ANSYS module. The comparisons of the energy density distribution along x, y=L/2 in laminate plate at several frequencies are presented in Fig.2. The EFEA method eliminates the volatility of solutions. The numerical results of EFEA agree well with the average value of the very dense FEA, and the results exhibit better agreement at higher frequencies. The EFEA results can capture the energy distribution level in the structure well using a much smaller number of elements than that of the conventional FEA. The difference of the results calculated by EFEA and FEA is within 1.6dB.

In order to analyze the complex structures, it is necessary to calculate the energy transmitted from the excitation location to the other components. The energy transmission coefficients for arbitrary joints need to be calculated and accounted for the coupling structures in the future work.



(a) 500Hz (b) 1000Hz

(c) 3000Hz (d) 5000Hz

Fig. 2. Distribution of energy density along *x*,*y*=L/2 in laminate plate (30 °/45 °/-45 °/-30 °) computed by EFEA and the very dense FEA (*f*=500Hz,1000Hz,3000Hz,5000Hz. Reference energy density1e-12*J*/*m*).

Summary

The EFEA formulation, using time and space averaged energy density as the primary variable, is developed for composite laminate plate structures in this paper. The composite laminate plates were assumed to be equivalent isotropic materials using the average concept. The EFEA method provides a practical approach to evaluate and predict the structural response at high frequencies, and goes beyond the limits of conventional FEA methods because of the computational cost or influences of uncertainty factors. The vibration response of the large scale composite laminate plate structures can be predicted in the design process and be designed to meet the increasingly demanding performance requirements.

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